

Trace Anomaly and Quasi-Particles in Finite Temperature $SU(N)$ Gauge Theory

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Abstract

We consider deconfined matter in $SU(N)$ gauge theory as an ideal gas of transversely polarized quasi-particle modes having a temperature-dependent mass $m(T)$. Just above the transition temperature, the mass is assumed to be determined by the critical behavior of the energy density and the screening length in the medium. At high temperature, it becomes proportional to T as the only remaining scale. The resulting (trace anomaly based) interaction measure $\Delta = (\epsilon - 3P)/T^4$ and energy density are found to agree well with finite temperature $SU(3)$ lattice calculations.

1 Introduction

The quark-gluon plasma in the region $T_c \leq T \leq 5 T_c$ presents a particularly challenging topic of investigation to strong interaction thermodynamics. The most suitable tool for such studies is the expectation value of the trace of the energy-momentum tensor, $\langle \Theta_\mu^\mu \rangle = \epsilon - 3 P$, which measures the deviation from conformal behavior and thus identifies the interaction still present in the medium. The aim of the present work is to study the temperature behavior of this measure and try to identify the underlying physics which causes it. The only *ab initio* calculations in the range of temperatures of interest here are obtained through finite temperature lattice QCD. In particular, pure $SU(3)$ gauge theory has been studied extensively, and through finite size scaling techniques the behavior is given in the continuum limit [1]. This case will therefore form the main basis of our study.

In Fig. 1a we show the temperature dependence of the energy and the pressure, divided by T^4 , for $SU(3)$ gauge theory, and in Fig. 1b that of the dimensionless interaction measure,

$$\Delta(T) = \frac{\langle \Theta_\mu^\mu \rangle}{T^4} = \frac{\epsilon - 3 P}{T^4}. \quad (1)$$

For all quantities, the extrapolation to the continuum limit is shown [1]. In the region around and just above T_c , the energy density rises much more rapidly than the pressure, leading to the observed rapid increase of $\Delta(T)$. Since asymptotically $\epsilon(T)/T^4$ and

$3 P(T)/T^4$ converge to their common Stefan-Boltzmann value ($8 \pi^2/15$), there must be some temperature T_p at which the growth rates change roles, with the pressure now increasing more rapidly. This leads to the peak observed for $\Delta(T_p \simeq 1.05 T_c)$, followed by a somewhat slower decrease. The transition itself is of first order [2, 3], as expected for a theory belonging to the Z_3 universality class [4].

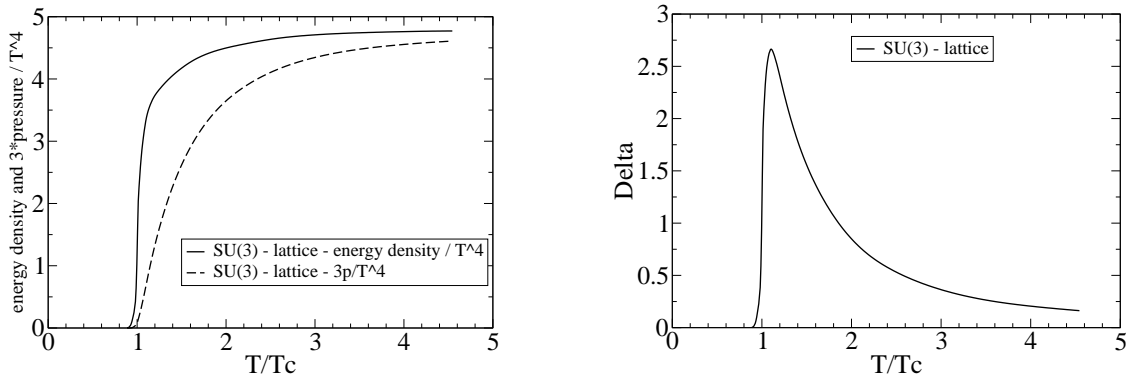


Figure 1: Temperature dependence (left) of the energy density and the pressure, and (right) of the interaction measure, for $SU(3)$ gauge theory [1].

Two further features of the interaction have become clear more recently. As shown in Fig. 2 (left), the decrease of Δ in the region under consideration here is in good approximation given by T^{-2} , so that $T^2 \Delta(T)$ becomes approximately constant very soon above T_c , and up to about $5 T_c$ [5]. Moreover, it is seen that $\Delta(T)$ in different $SU(N)$ theories scales very well with the number of gluonic degrees of freedom [6]; in other words, $\Delta(T)/(N^2 - 1)$ becomes a universal curve, as seen in Figure 2 (right).

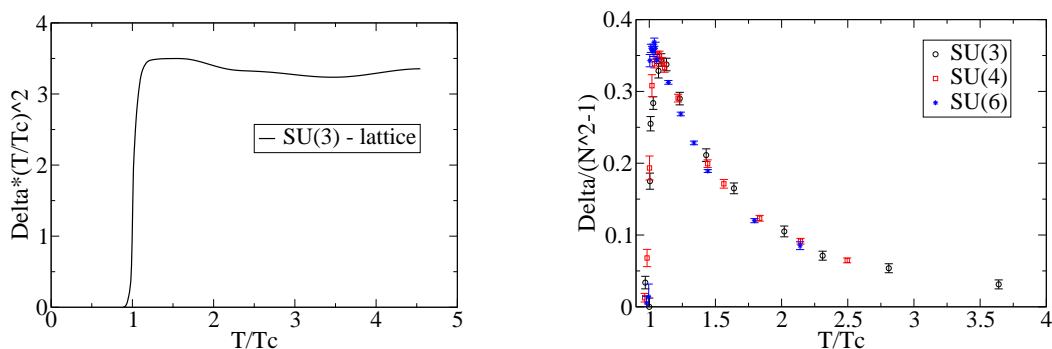


Figure 2: Temperature dependence (left) of $(T/T_c)^2 \Delta(T)$ for $SU(3)$ gauge theory [1] and (right) of the scaled interaction measure $\Delta(T)/(N^2 - 1)$ for $SU(N)$ gauge theories with $N = 3, 4, 6$.

In the following, we will first check to what extent any of the observed behavior can be accounted for by conventional or modified perturbation theory. Next we turn to the non-perturbative approach obtained through the relation of the interaction measure with the

gluon condensate and the corresponding bag pressure. Finally, in the main part of our work, we shall then show that a quasi-particle approach, based on massive gluonic modes, can indeed provide an excellent description of all the features observed in numerical finite temperature lattice studies of $SU(3)$ gauge theory .

2 Weak Coupling Approaches

In perturbation theory, the interaction measure for pure $SU(N)$ gauge theory is to leading order given by [1, 7]

$$\Delta_{\text{pert}} = \frac{(N^2 - 1)}{288} \frac{11}{12\pi^2} N^2 g^4(T), \quad (2)$$

with

$$N g^2(T) = \frac{24 \pi^2}{11 \ln(T/\Lambda_T)}. \quad (3)$$

The perturbative interaction measure thus does show the observed scaling in $N^2 - 1$ just mentioned, assuming $N g^2$ is kept constant. In eq. (3), Λ_T defines the lattice scale, which in the mentioned $SU(3)$ lattice studies [1] was found to be determined by $T_c/\Lambda_T = 7.16 \pm 0.25$. In this case, we thus obtain

$$\Delta_{\text{pert}} = \frac{11}{48\pi^2} g^4 = \frac{4\pi^2}{33} \frac{1}{\{\ln[7.16(T/T_c)]\}^2} \simeq \frac{1.2}{\{\ln[7.16(T/T_c)]\}^2}. \quad (4)$$

At $T/T_c = 3$, we have $\Delta_{\text{pert}} \simeq 0.13$, which is still about a factor 3 below the (continuum extrapolated) lattice result $\Delta_{\text{lat}} \simeq 0.4$. Hence at this temperature, leading order perturbation theory cannot yet reproduce the plasma interaction. Nevertheless, we have here $\alpha_s = g^2/4\pi \simeq 0.19$ for the strong coupling α_s , so that in principle perturbation theory seems to be applicable, and we could expect that at somewhat higher temperatures, above $T/T_c \simeq 5 - 10$, the perturbative form might account for the lattice result.

The evaluation of higher order perturbative terms has, however, shown that this is not the case. Infrared divergences in finite temperature field theory limit calculations to a finite order in the coupling g [8]; for the pressure, the highest perturbatively calculable order is g^5 , and calculations have now been extended to this order [9]. In Fig. 3, we show the result of expansions in different order g^n for the pressure in $SU(3)$ gauge theory, normalized to the Stefan-Boltzmann limit [10]. It is seen that in the temperature region of interest here, $T \leq 10 T_c$, the different orders lead to strong fluctuations; the final form, up to and including $O(g^5)$, still considerably undershoots the lattice results.

Moreover, for an understanding of the interaction effects, a comparison of lattice and perturbation theory results for the *pressure* is in fact quite misleading, since the major part of the pressure is given by the ideal gas component. To concentrate on just the interaction effects, we return to the interaction measure $\Delta(T)$, and here perturbation theory breaks down completely. The next-to-leading order (NLO) form for $SU(3)$ gauge theory,

$$\Delta_{\text{pert}} = \left(\frac{11}{8\pi^2} \right) \left[\frac{1}{6} g^4 - \frac{1}{\pi} g^5 \right] \quad (5)$$

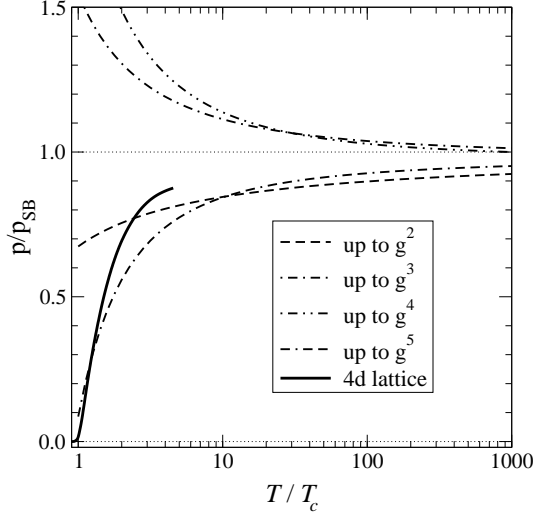


Figure 3: Perturbative expansions of the pressure in $SU(3)$ gauge theory [9,10], compared to the finite temperature lattice results [1].

becomes positive only for $g^2 \gtrsim 0.27$, which with the two-loop form of the coupling,

$$g^{-2} = \frac{11}{8\pi^2} \ln(T/\Lambda_T) + \frac{51}{88\pi^2} \ln[2 \ln(T/\Lambda_T)], \quad (6)$$

requires inconceivably high temperatures, above $10^6 T_c$. We conclude that the interaction of the plasma in the region of interest here, up to some $10 T_c$, must definitely require some non-perturbative features.

This situation has triggered numerous efforts to modify the perturbative approach to include such features. In one approach [10], the $O(g^6)$ term in the pressure is evaluated by a non-perturbative scale determination, using lattice results for magnetic screening. This leads to a systematic effective field theory, for which in principle all orders can be calculated. Another possibility is given by including sums over certain graph classes, thus effectively shifting the point about which the perturbation expansion is performed [11]. In particular, this is studied for the terms dominating at high temperature (hard thermal loops, HTL) [12,13] and leads to an improved convergence of the perturbation series of the pressure. Both approaches have in common

- a rather good description of the pressure for temperatures above 3 -5 T_c , but
- the range below about 3 T_c is still not well accounted for.
- Moreover, the strong order-by-order fluctuations for the interaction measure cause some doubt that the last order considered is really close to a “final” result.

To illustrate this, we show in Fig. 4 (left) the behavior obtained with the help of a partially non-perturbative $O(g^6)$ term [10], and in Fig. 4 (right) corresponding results from modified HTL calculations [12], in both cases compared to the form obtained in $SU(3)$ lattice QCD. The latter show for $\epsilon - 3P$ a decrease as $1/T^2$, so that $T^2\Delta(T)$ becomes approximately constant above T_c . We see in Fig. 4 (right) that in leading (LO) and next-to-leading order

(NLO) the breakdown of perturbation theory persists also in a HTL approach, and even the inclusion of a partially non-perturbative NNLO contribution cannot reproduce the lattice result, neither in size nor in functional form. Such a conclusion had been reached before, see e.g. [14]. Recent studies [15] have shown that in the case of full QCD with light quark flavors, the discrepancy between lattice data and weak-coupling results is reduced, with quite good agreement down to about $2 - 3 T_c$; however, neither the approximate T^{-2} behavior of $\Delta(T)$ in the range from T_c to about $5 T_c$, nor the sudden drop in the critical region can be thus obtained.

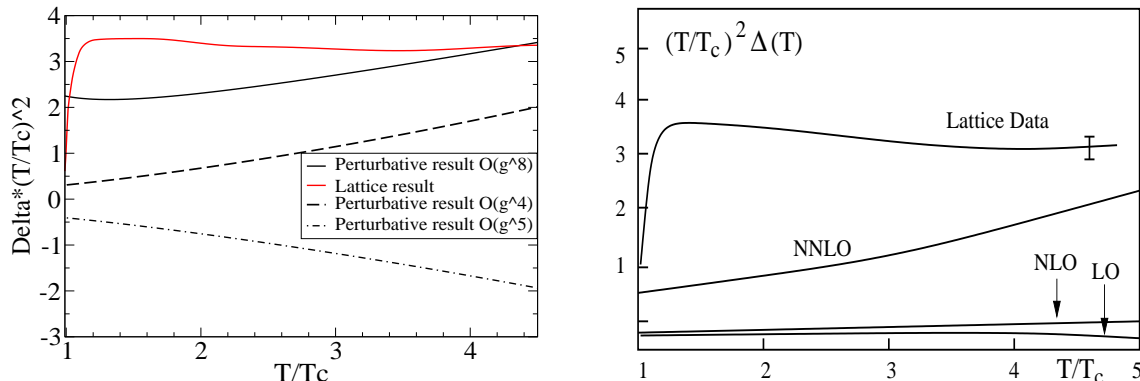


Figure 4: $(T/T_c)^2 \Delta(T)$ as predicted in systematic effective field theory (left, [10]) and in HTL resummed perturbation theory (right, [12]), compared to the continuum extrapolation of lattice studies [1].

In general, the breakdown observed in any perturbative treatment as we enter the transition region is of course not surprising. Critical or even pseudo-critical behavior with an increasing correlation range is simply not a perturbative phenomenon. We therefore have to find a non-perturbative approach to address the behavior of the plasma in this region.

3 Bag pressure and Gluon Condensate

One of the earliest attempts to account for the essential non-perturbative features is provided by the bag model [16, 17]. Here one implements confinement in an ideal gas picture by introducing a bag pressure, measuring the “level difference” between the physical vacuum and the ground state in the colored world of QCD. For the corresponding thermodynamics, this means that to the ideal gas partition function, $Z_0(T, V)$, a bag term is added,

$$T \ln Z_B(T, V) = T \ln Z_0(T, V) - BV \quad (7)$$

which in principle can be determined from a bag model description of hadron spectroscopy. The bag pressure simulates a form of interaction [18], as best seen by the resulting interaction measure,

$$\Delta(T) = \frac{4B}{T^4}. \quad (8)$$

We want to check here to what extent this is a viable description of the QGP interaction in the region above T_c , assuming B to remain temperature-independent.

The thermal expectation value of the trace of the energy-momentum tensor, $\langle \Theta_\mu^\mu \rangle = \epsilon - 3P$, is related to the gluon condensate, i.e., to the expectation value of gluon term in the QCD Lagrangian [19],

$$G^2 \equiv \frac{\beta(g)}{2g^3} G_{\mu\nu}^a G_a^{\mu\nu} = \frac{11N_c}{96\pi^2} G_{\mu\nu}^a G_a^{\mu\nu}, \quad (9)$$

where $G_{\mu\nu}^a = gF_{\mu\nu}^a$ is given by the gluon field of color a in the QCD Lagrangian. The last line of eq. (9) is obtained using the leading order perturbative beta function,

$$\beta(g) = \frac{11N_c}{48\pi^2} g^3 + O(g^5). \quad (10)$$

The value of $\langle G^2 \rangle = G_0^2$ at $T = 0$ has been estimated numerically, with $G_0^2 = 0.012 \pm 0.006 \text{ GeV}^4$ as “canonical” value [20]. In both analytical [19] and in lattice studies [1], $\epsilon - 3P$ is normalized to zero at $T = 0$, so that

$$\langle \Theta_\mu^\mu \rangle = \epsilon - 3P = G_0^2 - G_T^2, \quad (11)$$

where G_T^2 is the temperature-dependent gluon condensate. In the temperature range below T_c , we expect $G_T^2 = G_0^2$, so that $\epsilon - 3P = 0$. If the gluon condensate melts above T_c , the interaction measure becomes

$$\frac{\epsilon - 3P}{T^4} = \frac{G_0^2}{T^4}, \quad (12)$$

so that $B = G_0^2/4$ relates bag pressure and gluon condensate. The value for the latter given above leads to a bag pressure $B^{1/4} \simeq 230 \pm 30 \text{ MeV}$, which is in reasonable agreement with that obtained from hadron spectroscopy as given by the bag model.

The color summation in eq. (9) runs over the $N^2 - 1$ gluonic color degrees of freedom, so that we can write

$$G^2 = \frac{11N}{96\pi^2} g^2 \langle F_{\mu\nu}^a F_a^{\mu\nu} \rangle = \frac{11N}{96\pi^2} g^2 (N^2 - 1) \langle \bar{F}_{\mu\nu} \bar{F}^{\mu\nu} \rangle, \quad (13)$$

where $\langle \bar{F}_{\mu\nu} \bar{F}^{\mu\nu} \rangle$ denotes the gluon field contribution per color degree of freedom. The scaling of the interaction measure in $N^2 - 1$ observed for different $SU(N)$ theories is thus also in accord with the bag model dependence, keeping $g^2 N$ constant.

If one assumes that $G_T^2 = 0$ for $T \geq T_c$ ¹ the interaction measure becomes

$$\Delta(T) = \frac{G_0^2}{T^4} = \frac{G_0^2}{T_c^4} \left(\frac{T_c}{T} \right)^4 \simeq \frac{2.3}{(T/T_c)^4}, \quad (14)$$

using $T_c \simeq 0.27 \text{ GeV}$ for the $SU(3)$ deconfinement temperature. The lattice data are found to decrease much slower and are, as mentioned, in accord with a $1/T^2$ dependence.

¹It is known that such an assumption is in general not tenable [21]; a description in terms of a temperature-independent bag constant must therefore fail eventually. We want to determine here if it makes sense anywhere.

We therefore compare in Fig. 5 the results for $T^2\Delta(T)$ given by the lattice and the bag model forms. The bag model naturally cannot account for the structure immediately around T_c (the rise to the peak of $\Delta(T)$), but it also fails in the temperature region above T_c . Combining the bag model with some form of weak-coupling approach can somewhat improve the latter, but it can never reproduce the behavior in the critical region. This remains true also in various other, conceptually interesting attempts to modify the power of the T -dependence of $\Delta(T)$ [22–24].

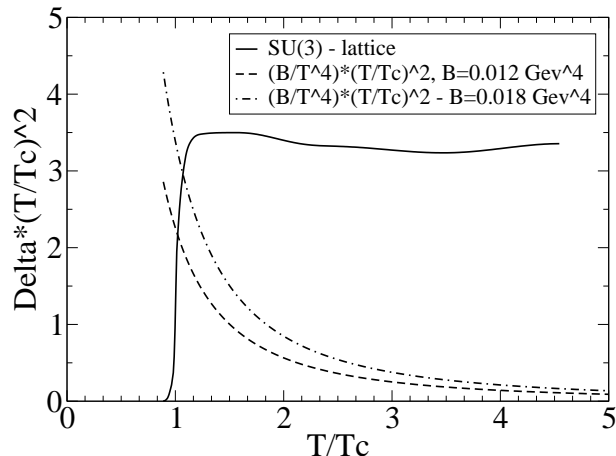


Figure 5: The temperature variation of $\Delta(T)(T/T_c)^2$ obtained from the bag pressure, compared to the corresponding lattice data [1].

4 The Quasi-Particle Approach

There thus remains the task to find a non-perturbative approach which takes into account the critical features arising in the temperature region in the range above T_c , as they were obtained in lattice studies. We stay in pure $SU(3)$ gauge theory, where, as mentioned, the extrapolation to the continuum limit is known [1]. The basis for our considerations here is the study of an ideal gas of constituents (“quasi-particles”) having dynamically or thermally generated masses [25–27]. The behaviour of an ideal gas of such massive gluon modes provides automatically the observed N_c^2 scaling and also leads to other features in accord with the functional behaviour found in $SU(N)$ gauge theories.

Interpretations of lattice QCD results in terms of a quasi-particle picture have been given in many versions [25–29]. In our approach, as in [25, 27], we shall include all interaction effects in a dynamically generated mass $m(T)$; most other studies maintain in addition a temperature-dependent bag constant. Instead, we want to relate the behavior near T_c to the critical behavior of the correlation length, causing the effective mass to increase as $T \rightarrow T_c$ from above [25].

The partition function of an ideal gas of constituents of mass $m(T)$ is in the Boltzmann

limit for $SU(N)$ gauge theory given by

$$\ln Z(T) = 2 \frac{(N^2 - 1)V}{2\pi^2} \int_0^\infty dp p^2 \exp\left(-\frac{1}{T}\sqrt{p^2 + m^2}\right) = 2 \frac{(N^2 - 1)VTm^2}{2\pi^2} K_2(m/T), \quad (15)$$

where $K_i(x)$ denotes the Hankel function of imaginary argument. The resulting pressure becomes

$$\begin{aligned} P(T) &= T \left(\frac{\partial \ln Z}{\partial V} \right)_T \\ &= 2 \frac{(N^2 - 1)T}{2\pi^2} \int_0^\infty dp p^2 \exp\left(-\frac{1}{T}\sqrt{p^2 + m^2}\right) = 2 \frac{(N^2 - 1)T^2 m^2}{2\pi^2} K_2(m/T) \end{aligned} \quad (16)$$

while the energy density is found to be

$$\begin{aligned} \epsilon(T) &= \frac{T^2}{V} \left(\frac{\partial \ln Z(T)}{\partial T} \right)_V \\ &= 2 \frac{(N^2 - 1)}{2\pi^2} \int_0^\infty dp p^2 \exp\left(-\frac{1}{T}\sqrt{p^2 + m^2}\right) \left\{ \sqrt{p^2 + m^2} - T \frac{m}{\sqrt{p^2 + m^2}} \left(\frac{dm}{dT} \right) \right\} \\ &= 2 \frac{(N^2 - 1)m^2 T^2}{2\pi^2} \left\{ 3K_2(m/T) + \left[\frac{m}{T} - \left(\frac{dm}{dT} \right) \right] K_1(m/T) \right\} \end{aligned} \quad (17)$$

In these expressions, we have maintained two spin degrees of freedom for the “massive” gluons; we return to this point shortly. Both energy density and pressure thus fall below the Stefan-Boltzmann limit, as is observed in the lattice data shown in Fig. 1. The resulting interaction measure is given by

$$\begin{aligned} \Delta(T) &= 2 \frac{(N^2 - 1)}{2\pi^2 T^4} \int_0^\infty dp p^2 \exp\left(-\frac{1}{T}\sqrt{p^2 + m^2}\right) \left\{ \sqrt{p^2 + m^2} - 3T - T \frac{m}{\sqrt{p^2 + m^2}} \left(\frac{dm}{dT} \right) \right\} \\ &= 2 \frac{(N^2 - 1)m^2}{2\pi^2 T^2} \left[\frac{m}{T} - \left(\frac{dm}{dT} \right) \right] K_1(m/T) \end{aligned} \quad (18)$$

If m is N -independent, the scaling in $N^2 - 1$ is evident. Moreover, if the effective mass m is linear in T , as in any conformal theory, $\Delta(T)$ vanishes. Given a running coupling, with $m^2 = N g^2(T) T^2/3$, we get

$$\Delta(T) = 2 \frac{(N^2 - 1)m^2}{12\pi^2 T^2} T \left(\frac{d(g^2 N)}{dT} \right). \quad (19)$$

We note, however, that such a “naive” quasi-particle description with finite masses seems to encounter a conceptual problem. Physical constituents of non-vanishing mass should have three, rather than two spin degrees of freedom, since a longitudinal polarization is excluded only for massless particles. The resulting changes in all thermodynamic quantities – e.g., the increase of the ideal gas energy density ϵ/T^4 from $8\pi^2/15$ to $12\pi^2/15$ – are definitely in disagreement with the observed high temperature lattice results. A simple shift to massive gluons thus cannot satisfactorily explain the interactions apparently still present in the high temperature gluon gas. More generally, a gauge invariant theory does

not allow massive physical gluons; the mechanism leading to effective thermal masses must thus be more subtle. The mentioned modified HTL perturbation theory approach, in which each order already includes some aspects of gluon dressing, not only leads to a rather rapid convergence of the expansion; here the contribution of longitudinal gluons vanishes in the limit $g \rightarrow 0$, so that one also obtains the right number of degrees of freedom for the Stefan-Boltzmann form [13]. Moreover, it has recently been argued [30] that massive gluons should in fact be transversely polarized, since two massless gluons cannot combine to form a longitudinally polarized massive gluon [31]. It thus seems justified to use the thermal mass scenario outlined above to address the temperature behaviour of the quark-gluon plasma.

The form of the effective mass entering in a quasi-particle approach description has been an enigma for quite some time. At sufficiently high temperature, T remains as the only scale, so that there we expect $m \sim T$. From perturbation theory one obtains in leading order for $SU(N)$ gauge theory a thermal screening mass $m^2 \sim N g^2(T) T/3$, but in view of the above mentioned difficulties, it seems best to leave the proportionality open. As we approach the critical point, perturbation theory in whatever form ceases to be applicable. We now have a medium of strongly interacting gluons, and the range of the forces between them becomes larger and larger as we approach the critical point. This range is governed by the correlation length, or in other words, by the distance up to which a given color charge can “see” other color charges. This distance is the QCD counterpart of the Debye screening radius in QED; we write it as $r_D(T) = 1/\mu(T)$, where $\mu(T)$ denotes the corresponding screening mass. It corresponds to the shift from $1/k^2$ to $1/(k^2 + \mu^2)$ experienced by the gluon propagator due to the presence of the medium.

The perhaps simplest view thus is to consider the mass of the quasi-gluon in the strongly coupled region to be the energy contained in a volume V_{cor} of the size defined by the correlation range,

$$m_{\text{crit}}(T) \sim \epsilon(T) V_{\text{cor}}(T). \quad (20)$$

In the case of a continuous transition, the critical part of the energy density becomes

$$\epsilon_{\text{crit}} \sim (t - 1)^{1-\alpha}, \quad (21)$$

where $t = T/T_c$ and α is the critical exponent for the specific heat. The correlation volume (for three space dimensions) can be written as

$$V_{\text{cor}} = 4\pi \int dr r^2 \Gamma(r, T), \quad (22)$$

where

$$\Gamma(r, T) \sim \frac{\exp\{-r/\xi(T)\}}{r^{1-\eta}}, \quad \xi(T) \sim (t - 1)^{-\nu} \quad (23)$$

specifies the correlation function Γ in terms of critical exponents ν for the correlation length $\xi(T)$ and η as anomalous dimension exponent. Combining these expressions and making use of the exponent equality relating α and ν , we obtain

$$m_{\text{crit}}(T) \sim (t - 1)^{1-\alpha-2\nu-\eta} = (t - 1)^{-(1+\eta-\nu)}; \quad (24)$$

for a continuous transition in three space dimensions. For $SU(2)$ gauge theory, the critical exponents are given by the corresponding exponents of the 3d Ising model [4], $\nu \simeq 0.63$ and $\eta \simeq 0.04$, suggesting

$$m_{crit}(T) \sim (t - 1)^{-0.41}. \quad (25)$$

This form is correct only in very near the critical point $t = 1$; for large temperatures, $\xi(t) \sim t$, so that the overall form expected for the mass of the quasi-gluon becomes

$$m(t) \simeq a(t - 1)^{-0.41} + bt, \quad (26)$$

where a and b are constants. The resulting behavior is illustrated in Fig. 6 (left). It would certainly be of interest to check this form directly through calculations in $SU(2)$ gauge theory; unfortunately, there does not seem to exist any lattice study providing an extrapolation to the continuum, thus eliminating finite lattice size effects. Older studies of $\epsilon(T)$ and $P(T)$ in terms of a gluon mass $m(T)$ [25] did in fact lead to the form shown in Fig. 26.

For $SU(3)$, the transition is of first order [2, 3], so that all quantities remain finite at T_c and an equivalent form cannot be given. Nevertheless, in all cases we have a strong increase of both $\epsilon(T)$ and $\Delta(T)$ in some range above T_c , and so we shall maintain the functional dependence (24/25) with an open exponent c . The resulting quasi-particle mass is thus expected to have the form

$$m(T) = \frac{a}{(t - 1)^c} + bt, \quad (27)$$

with constants a , b , c .

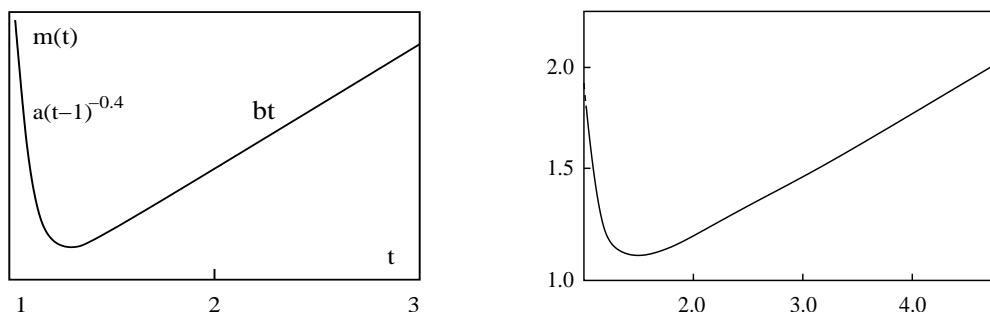


Figure 6: Left: Expected behavior of the effective quasi-particle mass $m(t)$ in (left) $SU(2)$ gauge theory, and (right) as obtained from a fit of lattice data in $SU(3)$ gauge theory.

Using this mass, we now determine the parameters a , b , c by calculating the energy density from eq. (17) and $\Delta(T)$ from eq. (19). The resulting mass and the corresponding parameters are shown in Fig. 6 (right). The fits to energy density and interaction measure are given in Fig. 7 and are seen to reproduce both quantities very well. We can thus conclude that the gluon plasma in $SU(3)$ gauge theory in the temperature region above T_c indeed behaves like a medium of quasi-particles with masses generated through non-perturbative thermal effects.

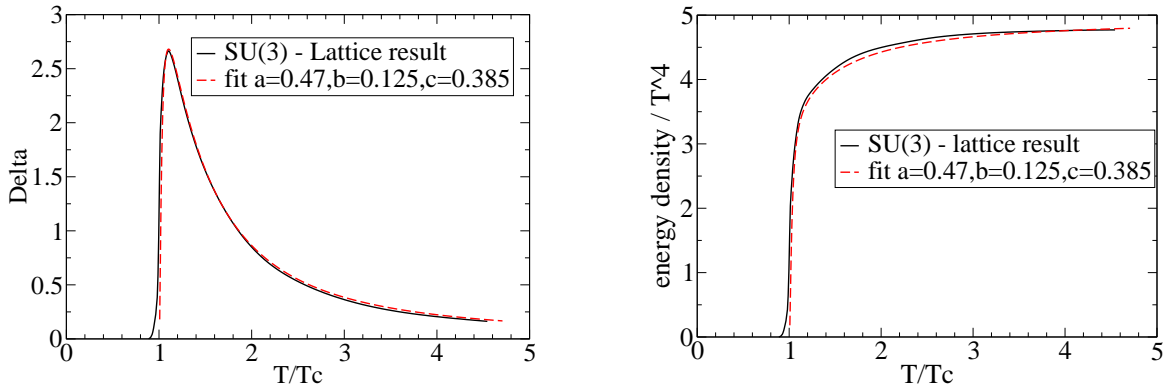


Figure 7: Interaction measure (left) and energy density (right) for $SU(3)$ gauge theory, compared to a quasi-particle description; the fitted constants a and b are given in GeV.

4.1 The Speed of Sound in the QGP

In a hadronic resonance gas, one finds [32] that the speed of sound drops to zero at the critical point defined by the limit of hadronic matter. It does so because any further energy increase goes into making more massive resonances, not into momentum and pressure. On the QGP side, in the quasi-particle just discussed, the behavior is very similar. As we lower the QGP temperature towards the confinement point, the increase of the quasi-particle mass has the same effect. In other words, a temperature increase above T_c lowers the mass and thus provides more momentum and pressure, causing an increase in the speed of sound.

The speed of sound, defined as

$$c_s^2 = \left(\frac{\partial p}{\partial \epsilon} \right)_V = \frac{s(T)}{C_V(T)}, \quad (28)$$

vanishes at T_c for a continuous transition, because the specific heat $C_V(T)$ diverges there, while the entropy density $s(T)$ remains finite. In the ideal gas limit, $s(T) \simeq 4 c_0 T^3$ and $C_V(T) \simeq 12 T^3$, so that $c_s^2 \rightarrow 1/3$. For the temperature-dependent mass (27), the speed of sound can be evaluated numerically, using eqns. (16) and (17). In Fig. 8, we show the resulting behavior obtained in our quasi-particle approach, in comparison to the $SU(3)$ lattice results. The two forms agree very well and in fact provide the behavior just indicated.

5 Conclusions

We have shown that the temperature behavior of the interaction measure defined by the trace anomaly of the energy-momentum tensor, $\Delta(T) = (\epsilon - 3P)/T^4$, is well described in terms of massive gluons with only transverse degrees of freedom. The gluon mass $m(T)$ increases sharply as $T \rightarrow T_c$ from above, due to the rapid growth of the correlation length in the critical region. On the other hand, with increasing temperatures and the approach

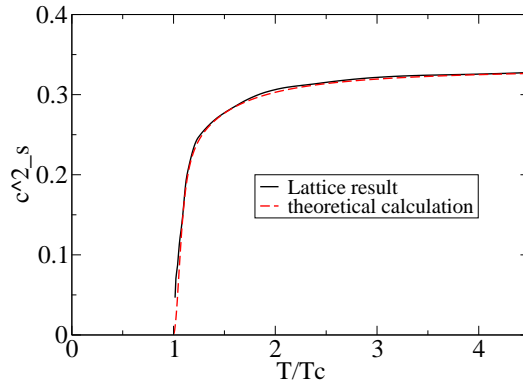


Figure 8: The speed of sound in the quasi-particle approach, compared to the $SU(3)$ lattice result [1].

to conformal behavior, $m(T) \sim T$. The combination of these two effects results in a minimum of $m(T)$ around $1.5 T_c$, signalling the transition from critical to hot gluon plasma. Even the hot plasma, however, remains strongly interacting; weak-coupling studies do not reproduce the observed lattice behavior in the region below about $5 T_c$.

6 Acknowledgement

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